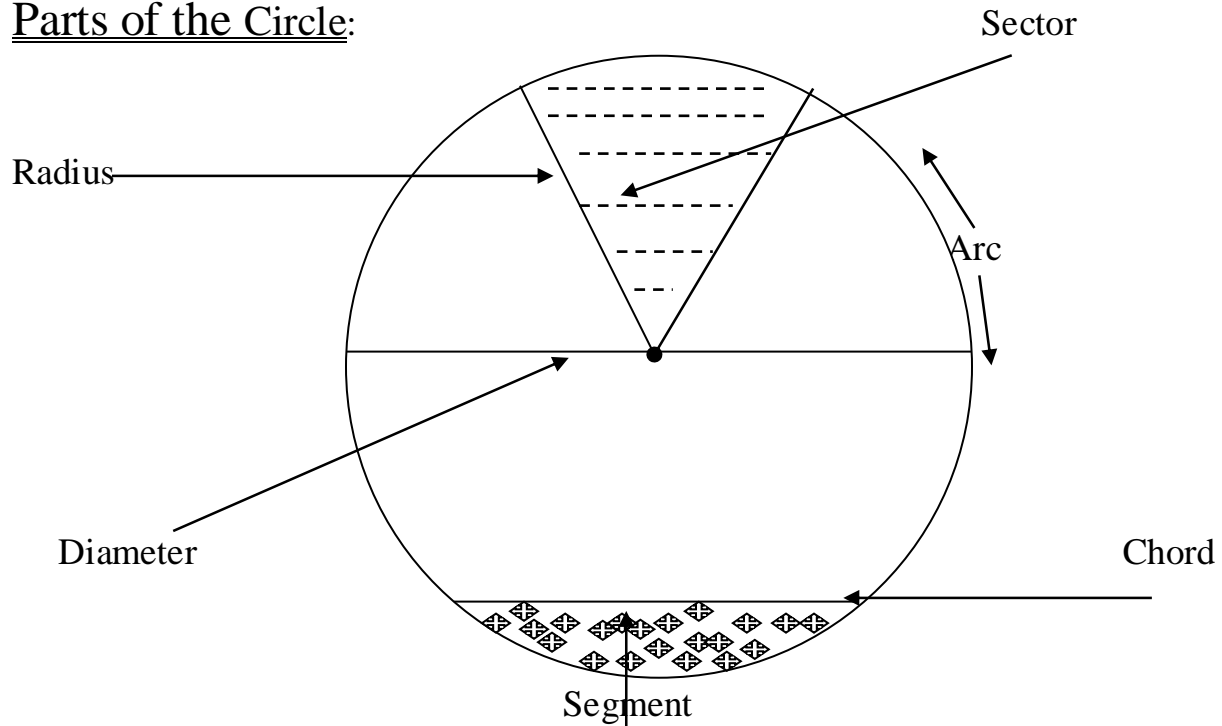


CHAPTER THIRTEEN

THE CIRCLE

Parts of the Circle:

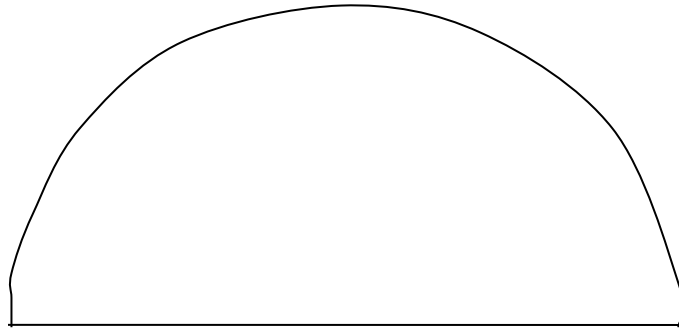


- 1) **The Circumference**: This is the distance around the circle.
- 2) **Chord**: This is a straight line which joins two points on the circumference.
- 3) **The diameter**: This is a special chord which passes through the centre of the circle.
- 4) **The radius**: This is a line drawn from the centre, to a point on the circumference.
- 5) **Arc**: This refers to a portion of the circumference.
- 6) **The segment**: This is the region between a chord and an arc.
- 7) **The sector**: This refers to the region between two radii.

Note:

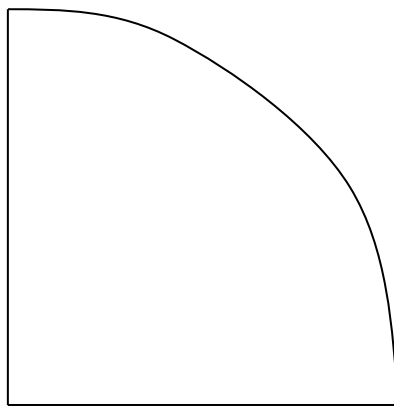
- i. For any circle, the radius $\times 2 =$ the diameter i.e. twice the radius gives us the diameter.

ii. Half a circle is referred to as a semi circle.



iii. A quadrant refers to one quarter of the area of a circle.

i.e.



iv. For a circle, $C = 2\pi r$, where C = the circumference, r = the radius and $\pi = 3.14$ or 3.142 or $\frac{22}{7}$.

Q1) A circle has a radius of 14cm. Determine the distance round it.

Solutions

$$C = 2\pi r \Rightarrow C = 2 \times 3.14 \times 14 = 88\text{cm.}$$

Q2) A city is circular in shape and its diameter is 30km. Determine the distance covered by a man, who walked twice round this city.

Solution

The distance covered by a man who did walk round the city once = the circumference.

$$D = 30\text{km}, \Rightarrow r = \frac{30}{2} = 15\text{km}.$$

$$C = 2\pi r, \Rightarrow C = 2 \times 3.14 \times 15 = 94\text{km}.$$

Distance covered by walking round the city twice = $2 \times 94 = 188\text{km}$.

Q3) A racing bike is travelling round a circular track whose radius is 40km, at a speed of 20km/h. Determine the time it will take to travel

- a) once round the track.
- b) thrice round the track.

Solution

(a) The distance covered by travelling once round the track = the circumference
 $= 2\pi r = 2 \times 3.14 \times 40 = 251\text{km}.$

a) the speed of racing bike = 20km/h

\therefore If 20km = 1 hour

$$\text{then } 251\text{km} = \frac{251}{20} \times 1 = 12.6\text{hrs}.$$

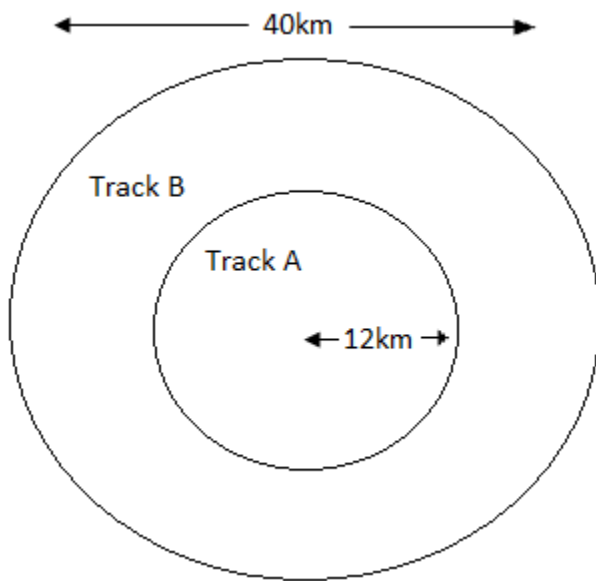
b) Distance travelled by travelling thrice round this track = $3 \times 251 = 753\text{km}.$

Speed of bike = 20km/h

If 20km = 1 hour

$$\text{Then } 753\text{km} = \frac{753}{20} \times 1 = 38\text{hrs}.$$

Q4)



Two cyclists, Addo and John are supposed to travel round two different circular tracks. Addo is to travel in track A at a speed of 40km/h and John is to travel in track B at a speed of 60km/h.

- Determine which of these men will be the first to complete his journey.
- Express the distance travelled by Addo as a fraction of the distance travelled by John.

Solution

The distance travelled by Addo = the circumference of track A = $2\pi r$
 $= 2 \times 3.14 \times 12 = 75\text{km}$.

Speed of Addo = 40km/h.

If 40km = 1 hour

$$\Rightarrow 75\text{km} = \frac{75}{40} \times 1 = 1.9.$$

\therefore Time taken by Addo to move round his track = 1.9 hrs.

Distance travelled by John = the circumference of track

$$B = 2\pi r = 2 \times 3.14 \times 20 = 126\text{km}.$$

Speed of John = 60km/h.

If 60km = 1 hour

$$\Rightarrow 126\text{km} = \frac{126}{60} \times 1 = 2.1$$

John will complete his journey in 2.1 hours..

Addo will finish first

b) Distance travelled by Addo = 75km and distance travelled by John = 126km

$$\text{Distance travelled by Addo as a fraction of that travelled by John} = \frac{75}{126} = \frac{25}{42}$$

Q5) A city which is circular in shape has a length of 420km. Determine the distance walked by Mr. Abu, if he walked from the centre of this city to a point on the city's boundary.

Solution

Length of the city = the circumference = 420km.

Distance travelled by Mr. Abu = the radius = ?

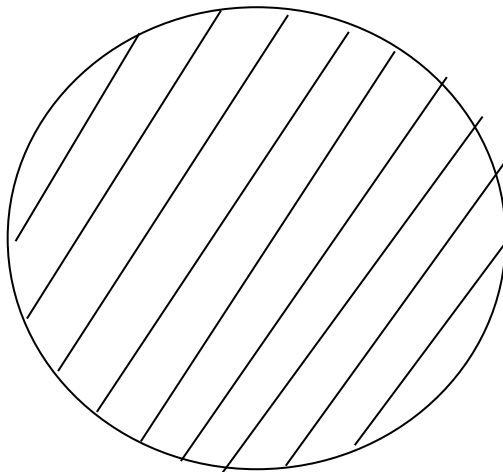
But since $C = 2\pi r \Rightarrow 420 = 2 \times 3.14 \times r$,

$$\Rightarrow 420 = 6.28r \Rightarrow r = \frac{420}{6.28},$$

$\Rightarrow r = 67$, \Rightarrow Distance walked by Mr. Abu = 67km.

The area of a circle:

The area of a circle refers to the region within the circle.



For example the shaded portion refers to the area of the given circle.

The area of a circle = πr^2 , where r = radius of the circle.

1) A circle has a radius of 7cm. Determine its area.

[Take $\pi = \frac{22}{7}$]

Solution

$$\text{Area} = \pi r^2 = \frac{22}{7} \times 7^2 = 154\text{cm}^2$$

2) A circular plot of land has an area of 255cm^2 .

Determine its diameter.

[Take $\pi = 3.142$]

Solution

$$\begin{aligned} \text{Since } A &= \pi r^2 \Rightarrow 255 = 3.14 \times r^2 \\ \Rightarrow \frac{255}{3.14} &= r^2 \Rightarrow 81 = r^2 \Rightarrow r = \sqrt{81} = 9\text{cm}. \end{aligned}$$

$$\text{Diameter} = 2r = 2 \times 9 = 18\text{cm}..$$

3) A man charges $\text{¢}2$ for weeding an area of 5m^2 . Determine how much he will charge if he weeds a circular field of radius 8m. [Take $\pi = 3.142$].

Solution

$$\text{Area of circular field to be weeded} = \pi r^2 = 3.142 \times 8^2 = 201\text{m}^2$$

$$\text{If } 5\text{m}^2 = \text{¢}2$$

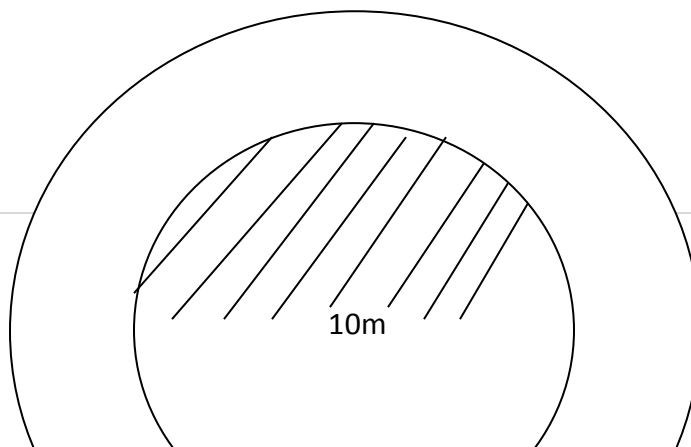
$$\Rightarrow 201\text{m}^2 = \frac{201}{5} \times 2 = 80, \Rightarrow \text{amount charged} = \text{¢}80.$$

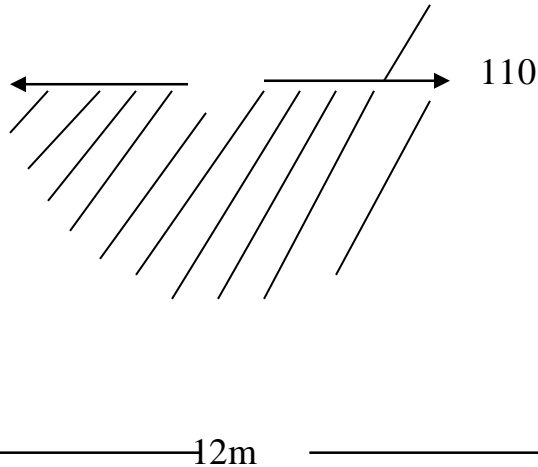
1. 4) A man owns a circular plot of land whose diameter is 12m. On a portion of this land which is circular in shape and of diameter 10m, he has planted onion.

Determine

- i. the fraction of the land on which the onion farm is located.
- ii. the percentage of the land on which the onion farm is located
- iii. the quantity of land left for future cultivation.

Solution





Let the shaded portion represent the onion farm. Since its diameter = 10m \Rightarrow radius = 5m.

The area of the onion farm = $\pi r^2 = 3.14 \times 5^2 = 78.5\text{m}^2$

Also since the diameter of the circular field = 12m \Rightarrow its radius = 6m.

The area of this field = $\pi r^2 = 3.14 \times 6^2 = 113\text{m}^2$

i) Fraction of the field on which the onion farm is located = $\frac{78.5}{113}$

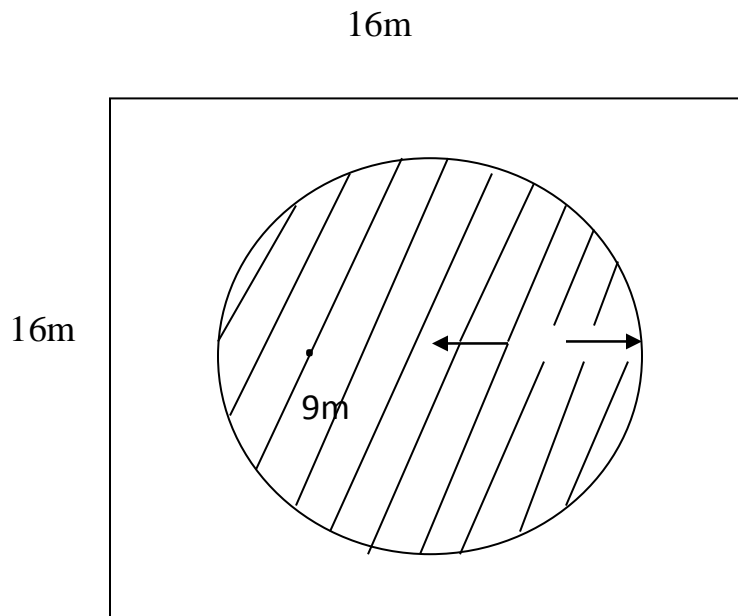
ii) The percentage of the land on which the onion farm is located = $\frac{78.5}{113} \times 100 = 69\%$.

iii) The portion of the land left for future cultivation = $113 - 78.5 = 34.5\text{m}^2$

(5) An onion farm which is circular in shape and of radius 9m is situated within a plot of land, which is in the shape of a square of side 16m.

Determine the fraction of the plot on which the onion farm is situated.

Solution



The area of the square plot = $16^2 = 256\text{m}^2$

The area of the shaded portion which represents the onion farm = $\pi r^2 = 3.14 \times 9^2 = 254\text{m}^2$.

The fraction of the plot on which the onion farm is located = $\frac{254}{256} = \frac{127}{128}$